- 1. A tunnel is drilled from one side of the Earth to the other, passing straight through the center. If the total mass of Earth is M_E , and its radius is R_E . If a particle of mass m is released into the tunnel, show that it will experience a simple harmonic motion and find its period. Hint: the force between the object and the earth is $|\vec{F}| = \frac{GM_Em}{r^2}$
- 2. A swinging door has mass m =0.2 kg and is attached to a spring with stiffness s =1.8 N/m. A dampener provide a frictional force -bv. The door position is given by x, and its equation of motions is:

$$m\ddot{x} + b\dot{x} + sx = 0$$

If a ball hits the door at t=0 with x=0 and v = 3m/s. Find the value of b such that the door:

- (a) Oscillate before closing, and determine its frequency of oscillation.
- (b) Find the maximum amplitude of the oscillations
- (c) Find the value of **b** such that the door close as quickly as possible
- 3. For damped harmonic oscillator system Show the following:
 - (a) The ratio of two successive maxima in the displacement of a damped harmonic oscillator is constant
 - (b) the period between successive zeros of a damped harmonic oscillator is constant, and is half the period between successive maxima
 - (c) If the amplitude of a damped harmonic oscillator decreases to 1/e of its initial value after $n \gg 1$ periods show that the ratio of the period of oscillation to the period of the oscillation with no damping is approximately $1 + \frac{1}{8\pi^2 n^2}$.
- 4. If the Q value is high show for damped harmonic oscillator show that the width of the displacement resonance curve is approximately $\frac{\sqrt{3}r}{m}$ where the width is measured between those frequencies where $X = X_{max}/2$.
- 5. A harmonic oscillator with mass m, natural angular frequency ω_0 , and damping constant **r** is driven by an external force $F(t) = F_0 cos(\omega t)$. Show that if $\omega = \omega_0$, then the instantaneous power supplied by the driving force is exactly absorbed by the damping force.